Exercice 1

A

a) $u = -3+3i \quad |u| = \sqrt{18} = 3\sqrt2 \quad \theta = \operatorname{Arg}(u) \begin{cases} \cos\theta = \frac{-3}{3\sqrt2} = -\frac{\sqrt2}{2} \\ \sin\theta = \frac{3}{3\sqrt2} = \frac{\sqrt2}{2} \end{cases} \Rightarrow \theta = \frac{3\pi}{4}$

$= 3\sqrt2\, e^{3i\pi/4}$

b) $z = \dfrac{6\sqrt2\, e^{i\pi/12}}{3\sqrt2\, e^{3i\pi/4}} = 2e^{i(\pi/12 - 3\pi/4)} = 2e^{i(-8\pi/12)} = 2e^{-2i\pi/3} = 2\left(-\frac12 - i\frac{\sqrt3}{2}\right) = -1 - i\sqrt3$

c) On a $e^{i\pi/12} = \dfrac{uz}{6\sqrt2} = \dfrac{(-3+3i)(-1-i\sqrt3)}{6\sqrt2} = \left(\dfrac{3+3\sqrt3}{6\sqrt2}\right) + i\left(\dfrac{-3+3\sqrt3}{6\sqrt2}\right) = \left(\dfrac{1+\sqrt3}{2\sqrt2}\right) + i\left(\dfrac{-1+\sqrt3}{2\sqrt2}\right)$

Donc $\cos\pi/12 = \dfrac{1+\sqrt3}{2\sqrt2} = \dfrac{\sqrt2+\sqrt6}{4}$ et $\sin\pi/12 = \dfrac{-1+\sqrt3}{2\sqrt2} = \dfrac{-\sqrt2+\sqrt6}{4}$

B

1) $\Delta = 64 - 4\times64 = -3\times64 < 0$ (2^d degré à coefficients réels)

$\Rightarrow$ 2 solutions complexes conjuguées $z_1 = \dfrac{8+8i\sqrt3}{2} = 4+4i\sqrt3$

$z_2 = \overline{z_1} = 4-4i\sqrt3$

2) $z_C = 8e^{-i\pi/3} = 8\left(\frac12 - i\frac{\sqrt3}{2}\right) = 4-4i\sqrt3$

$z_D = 8i\,e^{2i\pi/3} = 8i\left(-\frac12 + i\frac{\sqrt3}{2}\right) = -4\sqrt3 - 4i$

3) $OA = |z_A| = 8 \quad OB = |z_B| = 8 \quad OC = |z_A e^{-i\pi/3}| = |z_A| = 8 \quad OD = |z_B e^{2i\pi/3}| = |z_B| = 8$

donc, A B C D sont sur le cercle de centre O et de rayon 8

4) a) $z_1 = z_C - z_A = 4-4i\sqrt3 - 8 = -4-4i\sqrt3$

$z_2 = z_D - z_B = -4\sqrt3 - 4i - 8i = -4\sqrt3 - 12i$

donc $\sqrt3\, z_1 = \sqrt3(-4-4i\sqrt3) = -4\sqrt3 - 12i = z_2$

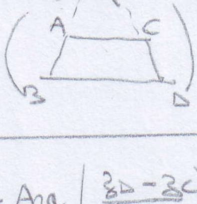
b) $z_3 = z_B - z_A = -8+8i \Rightarrow |z_3| = 8\sqrt2$

$z_4 = z_C - z_D = 4-4i\sqrt3+4\sqrt3+4i = (4+4\sqrt3)+i(4-4\sqrt3)$

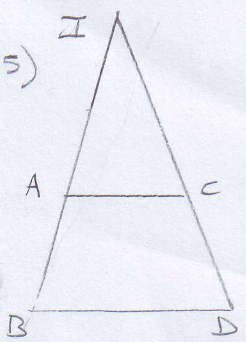
donc $|z_4| = \sqrt{(4+4\sqrt3)^2+(4-4\sqrt3)^2} = \sqrt{16+48+32\sqrt3+16+48-32\sqrt3} = \sqrt{128} = 8\sqrt2$ $(128 = 2\times64)$

c) $z_2 = z_1\sqrt3 \Rightarrow \overrightarrow{BD} = \sqrt3\,\overrightarrow{AC} \Rightarrow (BD)\parallel(AC)$ ($\overrightarrow{AC}$ et $\overrightarrow{BD}$ de même sens) $\Big\}\Rightarrow$ ABDC trapèze isocèle

$|z_3| = |z_4| \Rightarrow AB = DC$



5)



$(\overrightarrow{IB},\overrightarrow{ID}) = (\overrightarrow{AB},\overrightarrow{CD}) = \operatorname{Arg}\left(\dfrac{z_D - z_C}{z_B - z_A}\right)$ or $\dfrac{z_D - z_C}{z_B - z_A} = \dfrac{-z_4}{z_3} = \dfrac{(-4-4\sqrt3)+i(-4+4\sqrt3)}{-8+8i}$

$= \dfrac{(-1-\sqrt3)+i(-1+\sqrt3)}{2(-1+i)} = \dfrac{((-1-\sqrt3)+i(-1+\sqrt3))(-1-i)}{4} = \left(\dfrac{1+\sqrt3-1+\sqrt3}{4}\right) + i\left(\dfrac{1+\sqrt3+1-\sqrt3}{4}\right)$

$= \dfrac{\sqrt3}{2} + \dfrac12 i = e^{i\pi/6}$ donc $(\overrightarrow{IB},\overrightarrow{ID}) = \pi/6\ (2\pi)$

Exercice 2

1) $z_A = (1+i)(-2+2i)+2 = -2-2-2i+2i+2 = \boxed{-2}$

$z_A = (1+i)z_B + 2 \Rightarrow z_B = \frac{z_A - 2}{1+i} = \frac{-4+2i}{1+i} = \frac{(-4+2i)(1-i)}{2} = \boxed{-1+3i}$

2) a) $\underset{(w)}{\Omega} \to \underset{(w)}{\Omega} \Leftrightarrow w = (1+i)w + 2 \Leftrightarrow w(1-(1+i)) = 2 \Leftrightarrow w = \frac{2}{-i} = \boxed{2i}$

b) $z' - z = (1+i)z + 2 - z = iz + 2$ et $w - z = 2i - z$

or $(w-z)(-i) = (2i - z)(-i) = iz + 2 = z' - z$

donc $\boxed{\frac{z'-z}{w-z} = -i} \Leftrightarrow \begin{cases} \operatorname{Arg}\left(\frac{z'-z}{w-z}\right) = -\pi/2\,[2\pi] \\ \left|\frac{z'-z}{w-z}\right| = 1 \end{cases} \Leftrightarrow \begin{cases} (\overrightarrow{M\Omega}, \overrightarrow{MM'}) = -\pi/2\,[2\pi] \\ M'M = \Omega M \end{cases}$

$\Leftrightarrow$ $\Omega M M'$ triangle direct rectangle et isocèle en M (ce qui permet de construire M')

3) a) $M(z)$ tel que $|z+2-2i| = \sqrt{2} \Leftrightarrow AM = \sqrt{2} \Leftrightarrow M \in$ cercle Γ de centre A et de rayon $\sqrt{2}$

$(|z-(-2+2i)| = \sqrt{2})$

$AB = |-1+3i+2-2i| = |1+i| = \sqrt{2} \Rightarrow B \in \Gamma$

b) $z' + 2 = (1+i)z + 4$ et $(1+i)(z+2-2i) = (1+i)z + (1+i)(2-2i)$
$= (1+i)z + 4$

d'où $z' + 2 = (1+i)(z+2-2i)$

$M(z) \in \Gamma \Rightarrow |z+2-2i| = \sqrt{2}$

$\Rightarrow |z'+2| = |1+i||z+2-2i| = \sqrt{2} \times \sqrt{2} = 2$

$\Rightarrow M'(z') \in \Gamma'$ de centre le point d'affixe -2 qui est A et de rayon 2

(Autrement dit $F(\Gamma) \subset \Gamma'$)

Figure exercice 1

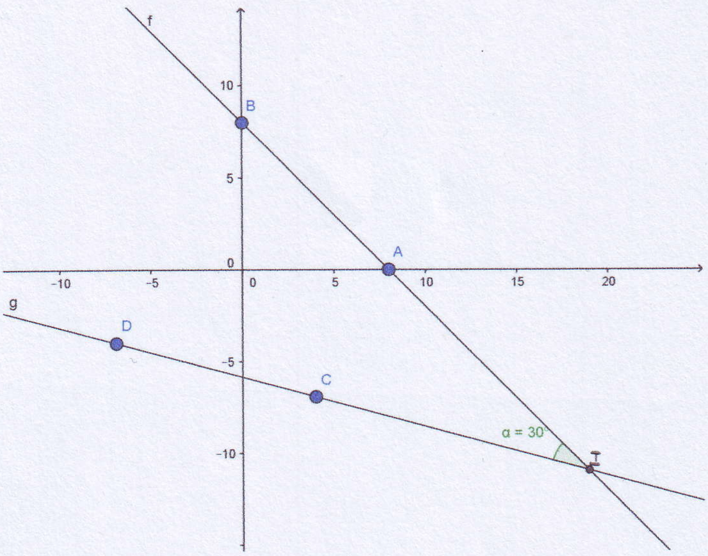


Figure exercice 2